MULTIMEDIA UNIVERSITY

STUDENT ID NO

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 2, 2017/2018**

## BMS1824 – MANAGERIAL STATISTICS

( All sections / Groups )

12 MARCH 2018
9.00 a.m. - 11.00 a.m.
(2 Hours)

---

**INSTRUCTIONS TO STUDENTS**

1. This question paper consists of **TWELVE (12)** printed pages with:
   **Section A:** Ten (10) multiple choice questions (20%)
   **Section B:** Three (3) structured questions (80%)

2. Answer **ALL** questions.

3. Answer **Section A** in the multiple-choice answer sheet provided and **Section B** in the answer booklet provided.

4. Students are allowed to use non-programmable scientific calculators with no restrictions.

## SECTION A: MULTIPLE CHOICE QUESTIONS (20 MARKS)

**There are TEN (10) questions in this section. Answer ALL questions on the multiple choice answer sheet.**

1. _____ refers to a process of data collection, summarizing and ways of describing data into useful information.

   A. Inferential statistics
   B. Descriptive statistics
   C. Frequency distribution
   D. Categorical data

2. Which of the following classification is NOT TRUE?

   A. Salaries of all staffs in a university [Population]
   B. Prices of 200 single storey houses selected from Banting, Selangor [Sample]
   C. Gender of 50% of subscribers to eight popular magazines [Population]
   D. Ages of rubber tappers from all estates in Malacca [Population]

3. Patients who received treatment at a private clinic were asked to indicate their ethnic background from a given list such as Malay, Chinese or Indian. This type of variable is classified as _____.

   A. Continuous
   B. Ordinal
   C. Nominal
   D. Discrete

4. The range of the middle 50% of the data is called the_____.

   A. Second quartile.
   B. Interquartile range.
   C. Variance.
   D. Mode.

5. Determine which statement is TRUE?

   A. A single, extremely large value can affect the median more than the mean.
   B. In a data set, the mode will always be unique.
   C. The height of basketball players is considered as a continuous variable.
   D. The positive square root of the variance is called as the coefficient of variation.

**Continued...**

6. Consider the following table that shows the details of the people applying for a particular job. The cell numbers indicate the number of applicants belonging to a particular age group and gender.

| | 25-40 years | 41-60 years |
|---|---|---|
| Male | 6 | 3 |
| Female | 5 | 7 |

What is the probability that a female or someone less than 41 years old applying for the job?

A. $\frac{4}{5}$
B. $\frac{23}{21}$
C. $\frac{6}{7}$
D. $\frac{5}{21}$

7. Linda is a goal shooter for her netball team. The probability of her scoring a goal is 0.85. In a particular game, Linda had 12 shots at goal. Determine the probability that she scored at least 11 goals.

A. 0.3012
B. 0.1422
C. 0.4434
D. 0.5566

8. The Poisson distribution might be used to model all of the following except _____.

A. the number of scratches on every three square feet of a car's body.
B. the number of customers that use an ATM during lunch hour
C. the number of car enter parking lot per hour
D. the number of job applicants who receive or not receive the job offer

**Continued...**

9. A large software development firm recently is interested in fostering good relations with their new local community and has encouraged their professional employees to engage in local service activities. They believe that the firm's professionals volunteer an average of more than 15 hours per month. If this is not the case, they will institute an incentive program to increase community involvement. The correct null and alternative hypotheses to test the belief of top management are

A. $H_0 : \mu \geq 15$ and $H_1 : \mu < 15$
B. $H_0 : \mu \leq 15$ and $H_1 : \mu > 15$
C. $H_0 : \mu = 15$ and $H_1 : \mu \neq 15$
D. $H_0 : \mu \neq 15$ and $H_1 : \mu = 15$

10. In regression analysis, the variable that is being predicted is
A. the independent variable
B. the dependent variable
C. usually denoted by X
D. none of the above

**Continued...**

## SECTION B: STRUCTURED QUESTIONS (80 Marks)

**There are THREE questions in this section. Candidates MUST answer ALL THREE questions.**

### Question 1 (25 Marks)

A manufacturer which produces fruit juices has separate department for packing process of their juices which stored in jars. For the packing task, they are using machines to pack cartons with jars. There is one old machine and one new machine has been using. The production manager interested to study on the times it takes for the machines to pack ten cartons each. The times taken (in seconds) are recorded as shown in the following table.

| New Machine | | | | | Old Machine | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 42.1 | 41.3 | 42.4 | 43.2 | 41.8 | 42.7 | 43.8 | 42.5 | 43.1 | 44.0 |
| 41.0 | 41.8 | 42.8 | 42.3 | 42.7 | 43.6 | 43.3 | 43.5 | 41.7 | 44.1 |

a) Compute the average times taken for new and old machines. Discuss the difference of the average values. [6 marks]

b) Compute the range and sample standard deviation of the times taken for both machines. [7 marks]

c) It is supposed that the new machine will pack faster on the average than the old machine. Do the data provide sufficient evidence to conclude that, on the average, the new machine packs faster? Perform test at the 0.05 level of significance. [12 marks]

### Question 2 (25 Marks)

a) The number of defects observed in the paint of a newly manufactured car is a discrete random variable. The probability distribution of this random variable is shown in the table below.

| Number of Defects | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Probability | 0.02 | 0.15 | 0.40 | 0.35 | 0.05 | 0.03 |

i. What would be the average number of defects per car? [4 marks]

ii. Compute the standard deviation. [5 marks]

**Continued...**

b) The cost of treatment per patient for a certain medical problem was modeled by one insurance company as a normal random variable with mean RM775 and standard deviation RM150.

i. What is the probability that the treatment cost of a patient is less than RM1,000 based on this model? [5 marks]

ii. What is the probability that the treatment cost of a patient is between RM600 to RM1100? [5 marks]

c) A study of 415 kindergarten students showed that they have watched television on average 5000 hours. The standard deviation of the population is 900 hours. Find the 95% confidence interval of the mean hours the kindergarten students watched television. [6 hours]

**Question 3 (30 Marks)**

a) An architect wants to determine the relationship between the heights (in feet) of a building and the number of stories in the building. Past records indicated that the height of a building depends on the number of stories it has. The data for the sample of 10 buildings in Kuala Lumpur are regressed and the summary output of the relationship between the two variables are shown below:

SUMMARY OUTPUT

| *Regression Statistics* | |
|---|---|
| Multiple R | 0.8044 |
| R Square | 0.6471 |
| Adjusted R Square | 0.6030 |
| Standard Error | 67.0041 |
| Observations | 10 |

ANOVA

| | *df* | *SS* | *MS* | *F* | *Significance F* |
|---|---|---|---|---|---|
| Regression | 1 | 65868.07 | 65868.07 | 14.67141 | 0.005016 |
| Residual | 8 | 35916.43 | 4489.554 | | |
| Total | 9 | 101784.5 | | | |

| | *Coefficients* | *Standard Error* | *t Stat* | *P-value* |
|---|---|---|---|---|
| Intercept | 222.9314516 | 104.9126 | 2.124926 | 0.066315 |
| Stories | 9.110383065 | 2.378487 | 3.830327 | 0.005016 |

**Continued...**

i. What are the dependent variable (y) and the independent variable (x) in the regression model above? [4 marks]

ii. State the estimated linear regression equation. [3 marks]

iii. What is the correlation coefficient for the regression model? Interpret the value. [3 marks]

iv. What is the coefficient of determination for the regression model? Interpret the value. [3 marks]

v. At 5% significance level, is there any significant relationship between the height of building and the number of stories of the building? Use p-value approach. [4 marks]

vi. Predict the height of a building if it has 35 stories. [3 marks]

b) A table below is consisting selected consumer goods for 2008 and 2015:

| Product | 2008 | | 2015 | |
|---|---|---|---|---|
| | Price (RM) | Quantity | Price (RM) | Quantity |
| Food | 340 | 115 | 345 | 90 |
| Rent | 600 | 16 | 600 | 13 |
| Drinks | 120 | 90 | 150 | 120 |
| Transportation | 250 | 43 | 260 | 56 |

Compute and interpret the Laspeyres Price Index (LPI) and Paashe Price Index (PPI) for 2015 using 2008 as the base period. [10 marks]

**End of Page**

## A. DESCRIPTIVE STATISTICS

Mean $(\bar{x}) = \dfrac{\sum_{i=1}^{n} X_i}{n}$

Standard Deviation $(s) = \sqrt{\dfrac{\sum_{i=1}^{n} X_i^2}{n-1} - \dfrac{(\sum_{i=1}^{n} X_i)^2}{n(n-1)}}$

Coefficient of Variation $(CV) = \dfrac{\sigma}{\bar{X}} \times 100$

Pearson's Coefficient of Skewness $(S_k) = \dfrac{3(\bar{X} - Median)}{s}$

## B. PROBABILITY

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A)× P (B) if A and B are independent

P (A | B) = P (A and B) ÷ P (B)

**Poisson Probability Distribution**

If $X$ follows a Poisson Distribution , $P(\lambda)$ where $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

then the mean = $E(X) = \lambda$ and variance = $VAR(X) = \lambda$

**Binomial Probability Distribution**

If $X$ follows a Binomial Distribution B($n$, $p$) where $P(X = x) = {}^nC_x p^x q^{n-x}$

then the mean = $E(X) = np$ and variance = $VAR(X) = npq$ where $q = 1 - p$

**Normal Distribution**

If $X$ follows a Normal distribution, $N(\mu, \sigma)$ where $E(X) = \mu$ and $VAR(X) = \sigma^2$

then $Z = \dfrac{X - \mu}{\sigma}$

## C. EXPECTATION AND VARIANCE OPERATORS

$E(X) = \Sigma[X \bullet P(X)]$

$VAR(X) = E(X^2) - [E(X)]^2$ where $E(X^2) = \Sigma[X^2 \bullet P(X)]$

If $E(X) = \mu$ then $E(cX) = c\,\mu$, $E(X_1 + X_2) = E(X_1) + E(X_2)$

If $VAR(X) = \sigma^2$ then $VAR(cX) = c^2\,\sigma^2$, $VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2) + 2\,COV(X_1, X_2)$

where $COV(X_1, X_2) = E(X_1 X_2) - [E(X_1)\,E(X_2)]$

## D. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

(100 – α) % Confidence Interval for Population Mean (σ Known) = $\mu = \overline{X} \pm Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Mean (σ Unknown) = $\mu = \overline{X} \pm t_{\alpha/2,n-1}\left(s/\sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Proportion $= \hat{p} \pm Z_{\alpha/2}\sigma_{p^\wedge}$ Where $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Sample Size Determination for Population Mean $= n \geq \left[\frac{(Z_{\alpha/2})\sigma}{E}\right]^2$

Sample Size Determination for Population Proportion $= n \geq \frac{(Z_{\alpha/2})^2\,\hat{p}(1-\hat{p})}{E^2}$

Where E = Limit of Error in Estimation

## E. HYPOTHESIS TESTING

**One Sample Mean Test**

| **Standard Deviation (σ) Known** | **Standard Deviation (σ) Not Known** |
|---|---|
| $Z = \frac{x-\mu}{\sigma/\sqrt{n}}$ | $t = \frac{x-\mu}{s/\sqrt{n}}$ |

**One Sample Proportion Test**

$z = \frac{\hat{p}-p}{\sigma_p}$ where $\sigma_p = \sqrt{\frac{p(1-p)}{n}}$

***Two Sample Mean Test***

**Standard Deviation (σ) Known**

$$z = \frac{(\overline{x_1}-\overline{x_2})-(\mu_1-\mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$$

**Standard Deviation (σ) Not Known**

$$t = \frac{(\overline{x_1}-\overline{x_2})-(\mu_1-\mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$$

where $S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1+n_2-2)}$

**Two Sample Proportion Test**

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p(1-p)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

where $p = \frac{X_1+X_2}{n_1+n_2}$

*where $X_1$ and $X_2$ are the number of successes from each population*

## F. REGRESSION ANALYSIS

**Simple Linear Regression**

**Population Model:** $Y = \beta_0 + \beta_1 X_1 + \varepsilon$

**Sample Model:** $\hat{y} = b_0 + b_1 x_1 + e$

**Correlation Coefficient**

$$r = \frac{\sum XY - \left[\frac{\sum X \sum Y}{n}\right]}{\sqrt{\left[\sum X^2 - \left(\frac{(\sum X)^2}{n}\right)\right]\left[\sum Y^2 - \left(\frac{(\sum Y)^2}{n}\right)\right]}} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

**ANOVA Table for Regression**

| Source | Degrees of Freedom | Sum of Squares | Mean Squares |
|---|---|---|---|
| Regression | *1* | SSR | MSR = SSR/1 |
| Error/Residual | $n-2$ | SSE | MSE = SSE/$(n-2)$ |
| Total | $n-1$ | SST | |

**Test Statistic for Significance of the Predictor Variable**

$t_i = \frac{b_i}{S_{b_i}}$ and the critical value = $\pm t_{\alpha/2,(n-p-1)}$

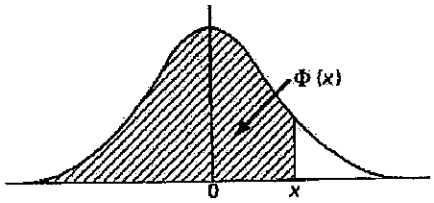
*Where p= number of predictor*

## G. INDEX NUMBERS

| | |
|---|---|
| **Simple Price Index** $P = \frac{p_t}{p_0} \times 100$ | **Laspeyres Quantity Index** $P = \frac{\sum p_0 q_t}{\sum p_0 q_0} \times 100$ |
| **Aggregate Price Index** $P = \frac{\sum p_t}{\sum p_0}(100)$ | **Paasche Quantity Index** $P = \frac{\sum p_t q_t}{\sum p_t q_0} \times 100$ |
| **Laspeyres Price Index** $P = \frac{\sum p_t q_0}{\sum p_0 q_0} \times 100$ | **Fisher's Ideal Price Index** $\sqrt{(Laspeyres\ Price\ Index)(Paa\ sche\ Price\ Index)}$ |
| **Paasche Price Index** $P = \frac{\sum p_t q_t}{\sum p_0 q_t} \times 100$ | **Value Index** $V = \frac{\sum p_t q_t}{\sum p_0 q_0} \times 100$ |

# TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to $x$. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.




| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0·00 | 0·5000 | 0·40 | 0·6554 | 0·80 | 0·7881 | 1·20 | 0·8849 | 1·60 | 0·9452 | 2·00 | 0·97725 |
| ·01 | ·5040 | ·41 | ·6591 | ·81 | ·7910 | ·21 | ·8869 | ·61 | ·9463 | ·01 | ·97778 |
| ·02 | ·5080 | ·42 | ·6628 | ·82 | ·7939 | ·22 | ·8888 | ·62 | ·9474 | ·02 | ·97831 |
| ·03 | ·5120 | ·43 | ·6664 | ·83 | ·7967 | ·23 | ·8907 | ·63 | ·9484 | ·03 | ·97882 |
| ·04 | ·5160 | ·44 | ·6700 | ·84 | ·7995 | ·24 | ·8925 | ·64 | ·9495 | ·04 | ·97932 |
| 0·05 | 0·5199 | 0·45 | 0·6736 | 0·85 | 0·8023 | 1·25 | 0·8944 | 1·65 | 0·9505 | 2·05 | 0·97982 |
| ·06 | ·5239 | ·46 | ·6772 | ·86 | ·8051 | ·26 | ·8962 | ·66 | ·9515 | ·06 | ·98030 |
| ·07 | ·5279 | ·47 | ·6808 | ·87 | ·8078 | ·27 | ·8980 | ·67 | ·9525 | ·07 | ·98077 |
| ·08 | ·5319 | ·48 | ·6844 | ·88 | ·8106 | ·28 | ·8997 | ·68 | ·9535 | ·08 | ·98124 |
| ·09 | ·5359 | ·49 | ·6879 | ·89 | ·8133 | ·29 | ·9015 | ·69 | ·9545 | ·09 | ·98169 |
| 0·10 | 0·5398 | 0·50 | 0·6915 | 0·90 | 0·8159 | 1·30 | 0·9032 | 1·70 | 0·9554 | 2·10 | 0·98214 |
| ·11 | ·5438 | ·51 | ·6950 | ·91 | ·8186 | ·31 | ·9049 | ·71 | ·9564 | ·11 | ·98257 |
| ·12 | ·5478 | ·52 | ·6985 | ·92 | ·8212 | ·32 | ·9066 | ·72 | ·9573 | ·12 | ·98300 |
| ·13 | ·5517 | ·53 | ·7019 | ·93 | ·8238 | ·33 | ·9082 | ·73 | ·9582 | ·13 | ·98341 |
| ·14 | ·5557 | ·54 | ·7054 | ·94 | ·8264 | ·34 | ·9099 | ·74 | ·9591 | ·14 | ·98382 |
| 0·15 | 0·5596 | 0·55 | 0·7088 | 0·95 | 0·8289 | 1·35 | 0·9115 | 1·75 | 0·9599 | 2·15 | 0·98422 |
| ·16 | ·5636 | ·56 | ·7123 | ·96 | ·8315 | ·36 | ·9131 | ·76 | ·9608 | ·16 | ·98461 |
| ·17 | ·5675 | ·57 | ·7157 | ·97 | ·8340 | ·37 | ·9147 | ·77 | ·9616 | ·17 | ·98500 |
| ·18 | ·5714 | ·58 | ·7190 | ·98 | ·8365 | ·38 | ·9162 | ·78 | ·9625 | ·18 | ·98537 |
| ·19 | ·5753 | ·59 | ·7224 | ·99 | ·8389 | ·39 | ·9177 | ·79 | ·9633 | ·19 | ·98574 |
| 0·20 | 0·5793 | 0·60 | 0·7257 | 1·00 | 0·8413 | 1·40 | 0·9192 | 1·80 | 0·9641 | 2·20 | 0·98610 |
| ·21 | ·5832 | ·61 | ·7291 | ·01 | ·8438 | ·41 | ·9207 | ·81 | ·9649 | ·21 | ·98645 |
| ·22 | ·5871 | ·62 | ·7324 | ·02 | ·8461 | ·42 | ·9222 | ·82 | ·9656 | ·22 | ·98679 |
| ·23 | ·5910 | ·63 | ·7357 | ·03 | ·8485 | ·43 | ·9236 | ·83 | ·9664 | ·23 | ·98713 |
| ·24 | ·5948 | ·64 | ·7389 | ·04 | ·8508 | ·44 | ·9251 | ·84 | ·9671 | ·24 | ·98745 |
| 0·25 | 0·5987 | 0·65 | 0·7422 | 1·05 | 0·8531 | 1·45 | 0·9265 | 1·85 | 0·9678 | 2·25 | 0·98778 |
| ·26 | ·6026 | ·66 | ·7454 | ·06 | ·8554 | ·46 | ·9279 | ·86 | ·9686 | ·26 | ·98809 |
| ·27 | ·6064 | ·67 | ·7486 | ·07 | ·8577 | ·47 | ·9292 | ·87 | ·9693 | ·27 | ·98840 |
| ·28 | ·6103 | ·68 | ·7517 | ·08 | ·8599 | ·48 | ·9306 | ·88 | ·9699 | ·28 | ·98870 |
| ·29 | ·6141 | ·69 | ·7549 | ·09 | ·8621 | ·49 | ·9319 | ·89 | ·9706 | ·29 | ·98899 |
| 0·30 | 0·6179 | 0·70 | 0·7580 | 1·10 | 0·8643 | 1·50 | 0·9332 | 1·90 | 0·9713 | 2·30 | 0·98928 |
| ·31 | ·6217 | ·71 | ·7611 | ·11 | ·8665 | ·51 | ·9345 | ·91 | ·9719 | ·31 | ·98956 |
| ·32 | ·6255 | ·72 | ·7642 | ·12 | ·8686 | ·52 | ·9357 | ·92 | ·9726 | ·32 | ·98983 |
| ·33 | ·6293 | ·73 | ·7673 | ·13 | ·8708 | ·53 | ·9370 | ·93 | ·9732 | ·33 | ·99010 |
| ·34 | ·6331 | ·74 | ·7704 | ·14 | ·8729 | ·54 | ·9382 | ·94 | ·9738 | ·34 | ·99036 |
| 0·35 | 0·6368 | 0·75 | 0·7734 | 1·15 | 0·8749 | 1·55 | 0·9394 | 1·95 | 0·9744 | 2·35 | 0·99061 |
| ·36 | ·6406 | ·76 | ·7764 | ·16 | ·8770 | ·56 | ·9406 | ·96 | ·9750 | ·36 | ·99086 |
| ·37 | ·6443 | ·77 | ·7794 | ·17 | ·8790 | ·57 | ·9418 | ·97 | ·9756 | ·37 | ·99111 |
| ·38 | ·6480 | ·78 | ·7823 | ·18 | ·8810 | ·58 | ·9429 | ·98 | ·9761 | ·38 | ·99134 |
| ·39 | ·6517 | ·79 | ·7852 | ·19 | ·8830 | ·59 | ·9441 | ·99 | ·9767 | ·39 | ·99158 |
| 0·40 | 0·6554 | 0·80 | 0·7881 | 1·20 | 0·8849 | 1·60 | 0·9452 | 2·00 | 0·9772 | 2·40 | 0·99180 |

# TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2·40 | 0·99180 | 2·55 | 0·99461 | 2·70 | 0·99653 | 2·85 | 0·99781 | 3·00 | 0·99865 | 3·15 | 0·99918 |
| ·41 | ·99202 | ·56 | ·99477 | ·71 | ·99664 | ·86 | ·99788 | ·01 | ·99869 | ·16 | ·99921 |
| ·42 | ·99224 | ·57 | ·99492 | ·72 | ·99674 | ·87 | ·99795 | ·02 | ·99874 | ·17 | ·99924 |
| ·43 | ·99245 | ·58 | ·99506 | ·73 | ·99683 | ·88 | ·99801 | ·03 | ·99878 | ·18 | ·99926 |
| ·44 | ·99266 | ·59 | ·99520 | ·74 | ·99693 | ·89 | ·99807 | ·04 | ·99882 | ·19 | ·99929 |
| 2·45 | 0·99286 | 2·60 | 0·99534 | 2·75 | 0·99702 | 2·90 | 0·99813 | 3·05 | 0·99886 | 3·20 | 0·99931 |
| ·46 | ·99305 | ·61 | ·99547 | ·76 | ·99711 | ·91 | ·99819 | ·06 | ·99889 | ·21 | ·99934 |
| ·47 | ·99324 | ·62 | ·99560 | ·77 | ·99720 | ·92 | ·99825 | ·07 | ·99893 | ·22 | ·99936 |
| ·48 | ·99343 | ·63 | ·99573 | ·78 | ·99728 | ·93 | ·99831 | ·08 | ·99896 | ·23 | ·99938 |
| ·49 | ·99361 | ·64 | ·99585 | ·79 | ·99736 | ·94 | ·99836 | ·09 | ·99900 | ·24 | ·99940 |
| 2·50 | 0·99379 | 2·65 | 0·99598 | 2·80 | 0·99744 | 2·95 | 0·99841 | 3·10 | 0·99903 | 3·25 | 0·99942 |
| ·51 | ·99396 | ·66 | ·99609 | ·81 | ·99752 | ·96 | ·99846 | ·11 | ·99906 | ·26 | ·99944 |
| ·52 | ·99413 | ·67 | ·99621 | ·82 | ·99760 | ·97 | ·99851 | ·12 | ·99910 | ·27 | ·99946 |
| ·53 | ·99430 | ·68 | ·99632 | ·83 | ·99767 | ·98 | ·99856 | ·13 | ·99913 | ·28 | ·99948 |
| ·54 | ·99446 | ·69 | ·99643 | ·84 | ·99774 | ·99 | ·99861 | ·14 | ·99916 | ·29 | ·99950 |
| 2·55 | 0·99461 | 2·70 | 0·99653 | 2·85 | 0·99781 | 3·00 | 0·99865 | 3·15 | 0·99918 | 3·30 | 0·99952 |

The critical table below gives on the left the range of values of $x$ for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3·075 | 0·9990 | 3·263 | 0·9994 | 3·731 | 0·99990 | 3·916 | 0·99995 |
| 3·105 | 0·9991 | 3·320 | 0·9995 | 3·759 | 0·99991 | 3·976 | 0·99996 |
| 3·138 | 0·9992 | 3·389 | 0·9996 | 3·791 | 0·99992 | 4·055 | 0·99997 |
| 3·174 | 0·9993 | 3·480 | 0·9997 | 3·826 | 0·99993 | 4·173 | 0·99998 |
| 3·215 | 0·9994 | 3·615 | 0·9998 | 3·867 | 0·99994 | 4·417 | 0·99999 |
| | | | 0·9999 | | 0·99995 | | 1·00000 |

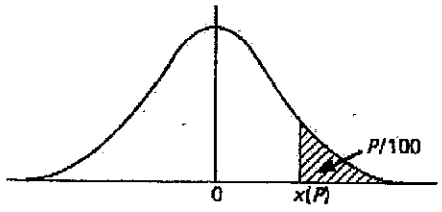
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

# TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If $X$ is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower $P$ per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



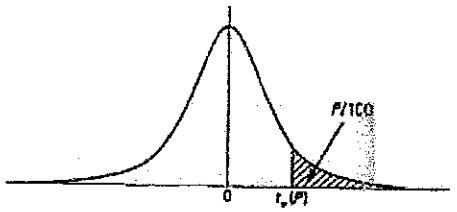

| $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 0·0000 | 5·0 | 1·6449 | 3·0 | 1·8808 | 2·0 | 2·0537 | 1·0 | 2·3263 | 0·10 | 3·0902 |
| 45 | 0·1257 | 4·8 | 1·6646 | 2·9 | 1·8957 | 1·9 | 2·0749 | 0·9 | 2·3656 | 0·09 | 3·1214 |
| 40 | 0·2533 | 4·6 | 1·6849 | 2·8 | 1·9110 | 1·8 | 2·0969 | 0·8 | 2·4089 | 0·08 | 3·1559 |
| 35 | 0·3853 | 4·4 | 1·7060 | 2·7 | 1·9268 | 1·7 | 2·1201 | 0·7 | 2·4573 | 0·07 | 3·1947 |
| 30 | 0·5244 | 4·2 | 1·7279 | 2·6 | 1·9431 | 1·6 | 2·1444 | 0·6 | 2·5121 | 0·06 | 3·2389 |
| 25 | 0·6745 | 4·0 | 1·7507 | 2·5 | 1·9600 | 1·5 | 2·1701 | 0·5 | 2·5758 | 0·05 | 3·2905 |
| 20 | 0·8416 | 3·8 | 1·7744 | 2·4 | 1·9774 | 1·4 | 2·1973 | 0·4 | 2·6521 | 0·01 | 3·7190 |
| 15 | 1·0364 | 3·6 | 1·7991 | 2·3 | 1·9954 | 1·3 | 2·2262 | 0·3 | 2·7478 | 0·005 | 3·8906 |
| 10 | 1·2816 | 3·4 | 1·8250 | 2·2 | 2·0141 | 1·2 | 2·2571 | 0·2 | 2·8782 | 0·001 | 4·2649 |
| 5 | 1·6449 | 3·2 | 1·8522 | 2·1 | 2·0335 | 1·1 | 2·2904 | 0·1 | 3·0902 | 0·0005 | 4·4172 |

# TABLE 10. PERCENTAGE POINTS OF THE $t$-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let $X_1$ and $X_2$ be independent random variables having a normal distribution with zero mean and unit variance and a $\chi^2$-distribution with $\nu$ degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's $t$-distribution with $\nu$ degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.




The limiting distribution of $t$ as $\nu$ tends to infinity is the normal distribution with zero mean and unit variance. When $\nu$ is large interpolation in $\nu$ should be harmonic.

| $P$ | 40 | 30 | 25 | 20 | 15 | 10 | 5 | 2·5 | 1 | 0·5 | 0·1 | 0·05 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu = 1$ | 0·3249 | 0·7265 | 1·0000 | 1·3764 | 1·963 | 3·078 | 6·314 | 12·71 | 31·82 | 63·66 | 318·3 | 636·6 |
| 2 | 0·2887 | 0·6172 | 0·8165 | 1·0607 | 1·386 | 1·886 | 2·920 | 4·303 | 6·965 | 9·925 | 22·33 | 31·60 |
| 3 | 0·2767 | 0·5844 | 0·7649 | 0·9785 | 1·250 | 1·638 | 2·353 | 3·182 | 4·541 | 5·841 | 10·21 | 12·92 |
| 4 | 0·2707 | 0·5686 | 0·7407 | 0·9410 | 1·190 | 1·533 | 2·132 | 2·776 | 3·747 | 4·604 | 7·173 | 8·610 |
| 5 | 0·2672 | 0·5594 | 0·7267 | 0·9195 | 1·156 | 1·476 | 2·015 | 2·571 | 3·365 | 4·032 | 5·893 | 6·869 |
| 6 | 0·2648 | 0·5534 | 0·7176 | 0·9057 | 1·134 | 1·440 | 1·943 | 2·447 | 3·143 | 3·707 | 5·208 | 5·959 |
| 7 | 0·2632 | 0·5491 | 0·7111 | 0·8960 | 1·119 | 1·415 | 1·895 | 2·365 | 2·998 | 3·499 | 4·785 | 5·408 |
| 8 | 0·2619 | 0·5459 | 0·7064 | 0·8889 | 1·108 | 1·397 | 1·860 | 2·306 | 2·896 | 3·355 | 4·501 | 5·041 |
| 9 | 0·2610 | 0·5435 | 0·7027 | 0·8834 | 1·100 | 1·383 | 1·833 | 2·262 | 2·821 | 3·250 | 4·297 | 4·781 |
| 10 | 0·2602 | 0·5415 | 0·6998 | 0·8791 | 1·093 | 1·372 | 1·812 | 2·228 | 2·764 | 3·169 | 4·144 | 4·587 |
| 11 | 0·2596 | 0·5399 | 0·6974 | 0·8755 | 1·088 | 1·363 | 1·796 | 2·201 | 2·718 | 3·106 | 4·025 | 4·437 |
| 12 | 0·2590 | 0·5386 | 0·6955 | 0·8726 | 1·083 | 1·356 | 1·782 | 2·179 | 2·681 | 3·055 | 3·930 | 4·318 |
| 13 | 0·2586 | 0·5375 | 0·6938 | 0·8702 | 1·079 | 1·350 | 1·771 | 2·160 | 2·650 | 3·012 | 3·852 | 4·221 |
| 14 | 0·2582 | 0·5366 | 0·6924 | 0·8681 | 1·076 | 1·345 | 1·761 | 2·145 | 2·624 | 2·977 | 3·787 | 4·140 |
| 15 | 0·2579 | 0·5357 | 0·6912 | 0·8662 | 1·074 | 1·341 | 1·753 | 2·131 | 2·602 | 2·947 | 3·733 | 4·073 |
| 16 | 0·2576 | 0·5350 | 0·6901 | 0·8647 | 1·071 | 1·337 | 1·746 | 2·120 | 2·583 | 2·921 | 3·686 | 4·015 |
| 17 | 0·2573 | 0·5344 | 0·6892 | 0·8633 | 1·069 | 1·333 | 1·740 | 2·110 | 2·567 | 2·898 | 3·646 | 3·965 |
| 18 | 0·2571 | 0·5338 | 0·6884 | 0·8620 | 1·067 | 1·330 | 1·734 | 2·101 | 2·552 | 2·878 | 3·610 | 3·922 |
| 19 | 0·2569 | 0·5333 | 0·6876 | 0·8610 | 1·066 | 1·328 | 1·729 | 2·093 | 2·539 | 2·861 | 3·579 | 3·883 |
| 20 | 0·2567 | 0·5329 | 0·6870 | 0·8600 | 1·064 | 1·325 | 1·725 | 2·086 | 2·528 | 2·845 | 3·552 | 3·850 |
| 21 | 0·2566 | 0·5325 | 0·6864 | 0·8591 | 1·063 | 1·323 | 1·721 | 2·080 | 2·518 | 2·831 | 3·527 | 3·819 |
| 22 | 0·2564 | 0·5321 | 0·6858 | 0·8583 | 1·061 | 1·321 | 1·717 | 2·074 | 2·508 | 2·819 | 3·505 | 3·792 |
| 23 | 0·2563 | 0·5317 | 0·6853 | 0·8575 | 1·060 | 1·319 | 1·714 | 2·069 | 2·500 | 2·807 | 3·485 | 3·768 |
| 24 | 0·2562 | 0·5314 | 0·6848 | 0·8569 | 1·059 | 1·318 | 1·711 | 2·064 | 2·492 | 2·797 | 3·467 | 3·745 |
| 25 | 0·2561 | 0·5312 | 0·6844 | 0·8562 | 1·058 | 1·316 | 1·708 | 2·060 | 2·485 | 2·787 | 3·450 | 3·725 |
| 26 | 0·2560 | 0·5309 | 0·6840 | 0·8557 | 1·058 | 1·315 | 1·706 | 2·056 | 2·479 | 2·779 | 3·435 | 3·707 |
| 27 | 0·2559 | 0·5306 | 0·6837 | 0·8551 | 1·057 | 1·314 | 1·703 | 2·052 | 2·473 | 2·771 | 3·421 | 3·690 |
| 28 | 0·2558 | 0·5304 | 0·6834 | 0·8546 | 1·056 | 1·313 | 1·701 | 2·048 | 2·467 | 2·763 | 3·408 | 3·674 |
| 29 | 0·2557 | 0·5302 | 0·6830 | 0·8542 | 1·055 | 1·311 | 1·699 | 2·045 | 2·462 | 2·756 | 3·396 | 3·659 |
| 30 | 0·2556 | 0·5300 | 0·6828 | 0·8538 | 1·055 | 1·310 | 1·697 | 2·042 | 2·457 | 2·750 | 3·385 | 3·646 |
| 32 | 0·2555 | 0·5297 | 0·6822 | 0·8530 | 1·054 | 1·309 | 1·694 | 2·037 | 2·449 | 2·738 | 3·365 | 3·622 |
| 34 | 0·2553 | 0·5294 | 0·6818 | 0·8523 | 1·052 | 1·307 | 1·691 | 2·032 | 2·441 | 2·728 | 3·348 | 3·601 |
| 36 | 0·2552 | 0·5291 | 0·6814 | 0·8517 | 1·052 | 1·306 | 1·688 | 2·028 | 2·434 | 2·719 | 3·333 | 3·582 |
| 38 | 0·2551 | 0·5288 | 0·6810 | 0·8512 | 1·051 | 1·304 | 1·686 | 2·024 | 2·429 | 2·712 | 3·319 | 3·566 |
| 40 | 0·2550 | 0·5286 | 0·6807 | 0·8507 | 1·050 | 1·303 | 1·684 | 2·021 | 2·423 | 2·704 | 3·307 | 3·551 |
| 50 | 0·2547 | 0·5278 | 0·6794 | 0·8489 | 1·047 | 1·299 | 1·676 | 2·009 | 2·403 | 2·678 | 3·261 | 3·496 |
| 60 | 0·2545 | 0·5272 | 0·6786 | 0·8477 | 1·045 | 1·296 | 1·671 | 2·000 | 2·390 | 2·660 | 3·232 | 3·460 |
| 120 | 0·2539 | 0·5258 | 0·6765 | 0·8446 | 1·041 | 1·289 | 1·658 | 1·980 | 2·358 | 2·617 | 3·160 | 3·373 |
| ∞ | 0·2533 | 0·5244 | 0·6745 | 0·8416 | 1·036 | 1·282 | 1·645 | 1·960 | 2·326 | 2·576 | 3·090 | 3·291 |